Complexity of Discrete Energy Minimization Problems

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Abstract

Energy minimization is NP-hard\textsuperscript{3}.
Is it approximable? Not yet resolved.
Sometimes yes: Potts, Metric, Logic MRF\textsuperscript{3}.
We prove that QPBO, planar energy with 3+ labels, and general energy minimization are all inapproximable.
Useful for algorithm design — finding “good” subclasses.
In practice, useful for model selection.

Complexity Axis & Main Results

Energy minimization problems vary greatly in approximation ratio.
Where do QPBO and general energy minimization fall on this axis?

Theorem: QPBO (binary labels) is complete in exp-APX.

Theorem: General energy minimization is complete in exp-APX.

Theorem: Planar energy with 3+ labels is complete in exp-APX.

Details

Non-deterministic Polynomial time Optimization (NPO)
The set of instances is recognizable in polynomial time.
The solution’s feasibility is verifiable in polynomial time.
A positive objective value.

Polynomial time Optimization (PO)
The problem is in NPO, and it is solvable in polynomial time.

Approximation-Preserving reduction (AP-reduction)
Reduce NPO problem $P_1$ to another NPO problem $P_2$.

Problem W3SAT-triv

INSTANCE: Boolean CNF formula $F$ with variables $x_1, ..., x_n$ and each clause assuming exactly 3 variables; non-negative integer weights $w_1, ..., w_n$.

SOLUTION: Truth assignment $\pi$ to the variables that either satisfies $F$ or assigns the trivial, all-true assignment.

MEASURE: $\min \sum w_i \tau(\pi)$.

References