Efficient Continuous Relaxations for Dense CRF

Alban Desmaison Rudy Bunel Pushmeet Kohli Philip H.S. Torr
M. Pawan Kumar

Department of Engineering Science, University of Oxford Microsoft Research

Introduction

Dense-CRF:

- Popular framework for vision problem modelisation.
- MAP estimation problem is of main interest.
- Allow modelisation of long range interactions.

Main challenge:

- Methods for sparse CRF does not scale for dense ones.
- Implementation requires $\mathcal{O}(n^2)$ computations.

Related work

MF based methods

Krähenbühl and Koltun initial work. [Kra. 2011]

- Use MF to approximate the MAP problem.
- Restricted to Gaussian pairwise terms.
- Use Permutohedral Lattice for $\mathcal{O}(n)$ computations.

SDP based method

[Wang 2015]

- SDP relaxation of the MAP problem.
- Use low-rank approximation to make it tractable.

Preliminaries

Energy function

$$E(\mathbf{x}) = \sum_{a=1}^{N} \phi_a(x_a) + \sum_{a=1}^{N} \sum_{\substack{b=1 \\ b \neq a}}^{N} \psi_{a,b}(x_a, x_b).$$

Where $\phi_a(x_a)$ represent the unary potentials for a given point a and $\psi_{a,b}(x_a, x_b)$ represents the pairwise interaction between a and b.
The considered pairwise here are gaussian pairwise terms:

$$\psi_{a,b}(i,j) = \mu(i,j) \sum_m w^{(m)} k(\mathbf{f}_a^{(m)}, \mathbf{f}_b^{(m)}),$$

Permutohedral Lattice

[Adams 2010]

Allows to compute the following in $\mathcal{O}(n)$ complexity (instead of $\mathcal{O}(n^2)$):

$$\forall a \in [1, N], \quad v'_a = \sum_{b=1}^{N} k(\mathbf{f}_a, \mathbf{f}_b) v_b,$$

Motivation

For sparse CRFs, relaxation-based methods are both efficient and have strong guarantees.
But these methods does not scale with the number of pairwise connections. Thus the only feasible algorithm for dense CRF MAP estimation is the MF-based approximation.
Here **we present efficient algorithms to solve convex relaxations of the original MAP estimation problem**. We thus present the first efficient algorithms with guarantees for the MAP estimation problem for Dense CRFs.

QP relaxations

Convex QP relaxation

- Conditional gradient and optimal step size in $\mathcal{O}(n)$.
- Convex problem not sensible to initialisation.

Generic difference of convex relaxation

- Use concave convex procedure iteratively.
- Each step of the CCCP requires solving a convex QP as above.
- Monotonic decrease to a local minima.

Specialised difference of convex

- Works only for negative semi-definite pairwise potentials.
- Use concave convex procedure iteratively.
- The convex QP is very easy to solve.
- Fast monotonic decrease to a local minima.

LP relaxation

LP relaxation of the MAP estimation problem

[Kleinberg 2002]

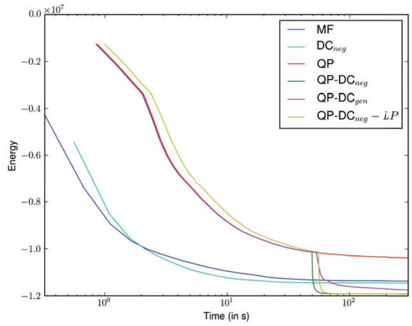
- Best possible relaxation of the original problem.
  - Tight in the 2 label case.
  - 2-approximation for more than 2 labels.
- Can compute the subgradient in $\mathcal{O}(n log(n))$.
- Use a divide and conquer approach based on the permutohedral lattice algorithm.
  - Do not compute full matrix vector product.
  - Consider triangular matrix vector product.
  - Perform divide and conquer to always consider full blocks.
- Reach the unique optimal solution via projected subgradient descent.

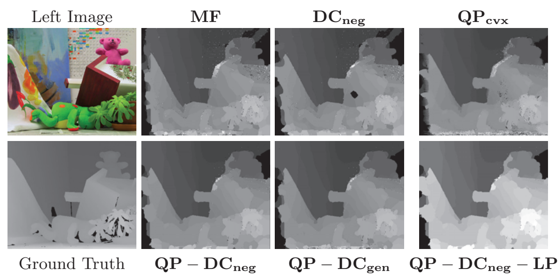
Results

The convex relaxation-based methods presented here reliably lead to better energy compared to the MF-based approach.

Stereo Matching

Evolution of the energy with respect to time for the Teddy image and the final matchings:





Semantic Segmentation