Similarity Registration Problems for 2D/3D Ultrasound Calibration

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UCL

Introduction

Ultrasound (US) is widely used in medical diagnostics and during therapy as a low-cost, flexible, and real-time imaging technique. In fetal medicine, US is both used for non-invasive diagnostics and to guide surgical interventions.

Pose tracking and calibration of a 3D US probe enhances its applications to Computer Assisted intervention (CAI) in the following aspects:

- Building large and detailed 3D models from several 3D US frames.
- Enabling freehand 4D US, i. e. registering both a 3D volume and its temporal evolution in a single coordinate system while the probe is being freely moved.
- Registration between 3D US data and other instruments, such as biopsy needles.

Calibrating a tracked US probe is done by scanning a calibration target with known shape. When using a curvilinear US probe, this is a **similarity registration problem** (also known as Pose-and-Scale problem), and it involves estimating both a rigid transformation and a scale factor that converts US scan units to metric coordinates.

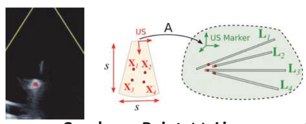
In this paper we use a tracked needle as the calibration target (a linear object) and the calibration problem becomes the **similarity registration between two sets of 3D lines** (using a 3D US probe), or the **similarity registration between co-planar points and 3D lines** (using a 2D US probe).

Formulation

SCANNING A TRACKED NEEDLE

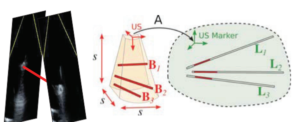
A needle is measured as a point in a 2D US, and as a line segment in a 3D US.

2D US



Co-planar Point ↔ Line

3D US

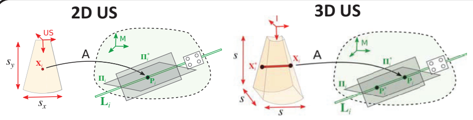


Line ↔ Line

CONVERTING TO (POINT ↔ PLANE) CORRESPONDENCES

Both problems are converted to the similarity registration between 3D points and 3D planes by defining each line Li as two planes, and each line Bi as two points Xi, Xi*.

2D US **3D US**



2 linear constraints per acquisition

$$\Pi_i^T A X_i = 0$$
$$\Pi_i^{*T} A X_i = 0$$

4 linear constraints per acquisition

$$\Pi_i^T A X_i = 0 \quad \Pi_i^T A X_i^* = 0$$
$$\Pi_i^{*T} A X_i = 0 \quad \Pi_i^{*T} A X_i^* = 0$$

5 independent quadratic constraints

$$S^T S = S S^T = \begin{pmatrix} s^2 & 0 & 0 \\ 0 & s^2 & 0 \\ 0 & 0 & s^2 \end{pmatrix}$$

GENERAL SOLUTION

The general problem has 13 unknown linear parameters and it can be **linearly solved** from **6 point-line correspondences** or **3 line-line correspondences**. However, A has 5 independent quadratic constraints, and thus it can be **minimally solved** from **4 point-line correspondences** or **2 line-line correspondences**.

Linear system **N X 13** $\begin{pmatrix} vec(S) \\ t \\ 1 \end{pmatrix} = 0$

SVD

$$A = aA_a + bA_b + cA_c + dA_d + eA_e + fA_f$$

Polynomial system **5 X 13** $\begin{pmatrix} b^3 \\ ab^2 \\ be \\ bd \\ bc \\ b^2 \\ ab \\ e \\ d \\ c \\ b \\ a \\ 1 \end{pmatrix} = 0$

SIMPLIFIED SOLUTION (CO-PLANAR POINT ↔ LINE)

When the **point correspondences are co-planar** (2D US), even though the general solution works, the problem can be simplified to 10 unknown linear parameters and it can be **linearly solved** from **5 point-line correspondences.**

$$\Pi_i^T \bar{A} (x_i\ y_i\ 1)^T = 0$$
$$\Pi_i^{*T} \bar{A} (x_i\ y_i\ 1)^T = 0$$

$$\bar{A} = \begin{pmatrix} \bar{S} & t \\ 0 & 1 \end{pmatrix} \qquad \bar{S}\bar{S}^T = \begin{pmatrix} s^2 & 0 \\ 0 & s^2 \end{pmatrix}$$

Linear system **N X 10** $\begin{pmatrix} vec(\bar{S}) \\ t \\ 1 \end{pmatrix} = 0$

SVD

$$\bar{A} = a\bar{A}_a + b\bar{A}_b + c\bar{A}_c$$

Polynomial system **2 X 6** $\begin{pmatrix} ab^2 \\ b^2 \\ ab \\ b \\ a \\ 1 \end{pmatrix} = 0$

CALIBRATION ALGORITHMS

3line3D: General linear solution. Equivalent to [13], [27].

2line3D: General minimal solution.

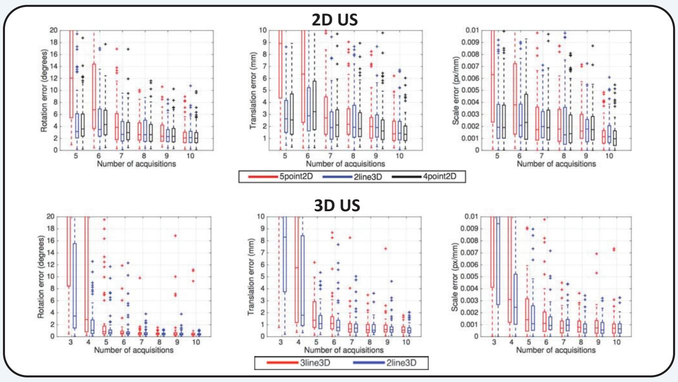
5point2D: Simplified linear solution for co-planar points.

4point2D: Simplified minimal solution for co-planar points. A particular case of [19].

Results

SYNTHETIC DATA

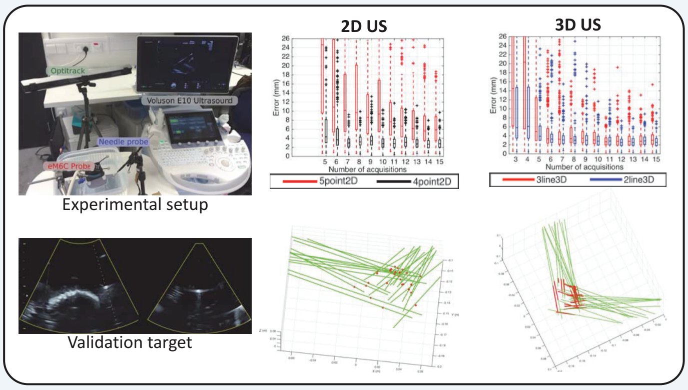
Our method is tested in a virtual environment where a fixed 2D/3D US probe scans a tracked needle at 30 different random positions. The US probe has a depth range of 107mm, angle range [−50°, 50°], a scale factor (s = 0.24 mm/px). Both the US scans and the needle tracking measurements are injected with Gaussian noise (σ = 2 pixel, σ = 1mm respectively). The results are compared against groundtruth values of rotation, translation, and scale factor.



REAL DATA

Our algorithms are tested using a GE Voluson E10 machine with a eM6C probe (3D US). The scanning depth is set to 107mm, and both 3D US and 2D US data are obtained with the same equipment and settings. Needle tracking is achieved with Optitrack V120 Trio. The US measurements are performed in a container filled with water at room temperature.

After 20 calibration trials, we scan a 3D point target and measure its projection reconstruction accuracy (PRA), i. e., the difference in mm between the 3D point location measured using the needle tip and its projection from the US scan. We performed 10 acquisitions of the 3D point target. Each distribution contains 200 error measurements.



CONCLUSIONS

Linear and minimal solutions are tested to calibrate a US probe using a tracked needle with both 3D and 2D data. This is useful in medical imaging to guide a biopsy needle in US based interventions. The method can be easily extended to additional US calibration problems using other types of calibration targets, e. g. scanning single plane target leads to the similarity registration between co-planar lines and 3D planes (2D US) or between two sets of 3D planes (3D US). In other computer vision domains this algorithm can potentially be used as an extension of the pose-and-scale problem to the alignment of line-based and/or plane-based SfM sequences.

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References

[13] Vasconcelos et. al. Spatial calibration of a 2d/3d ultrasound using a tracked needle. IJCARS 2016

[19] Ventura et. al. A minimal solution to the generalized pose-and-scale problem. CVPR 2014

[27] Bartoli, Sturm The 3d line motion matrix and alignment of reconstructions. CVPR 2001

[28] Ramalingam et. al. P2π: A minimal solution for registration line of 3d points to 3d planes. ECCV 2010

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