Resonant Deformable Matching: Simultaneous Registration and Reconstruction

John Corring, Anand Rangarajan

University of Florida

A New Idea for Representing Curves and Surfaces

Deformable registration techniques applied to point-sets often do not reflect the connectedness of the underlying shape. With the *complex wave representation*,

$$\psi_S(x) = \sum_{(m,\nu)\in S} \exp\{-\frac{||x-m||^2}{2\sigma^2} + i\frac{\nu\cdot(x-m)}{\lambda}\}, \quad (1)$$

we can add this feature (respecting the connectedness of a shape) to a registration pipeline that uses only normal- and point-based distance evaluation.

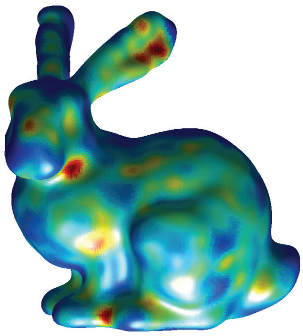

Figure 1: The surface is recovered using marching cubes on ψ_S. Then it is painted with relative intensities from $|\psi_S|^2$.

Properties of the Complex Wave Representation

Let g be the isotropic Gaussian:

- The representation **distinguishes** oriented point-sets: ψ. is injective from $(\mathbb{R}^d \times S^{d-1})^*$ into $L^2(\mathbb{R}^d)$.
- It provides **continuation** of the normal field: $\nabla\theta_\psi$ is defined a.e. with isolated (possible) singularities near Voronoi boundaries.
- It provides approximately **linear composition**: Given oriented point-sets S, T, $\psi_{S\cup T} \approx \psi_S + \psi_T$.
- Given two oriented points satisfying certain basic conditions, the phase is continuous through a *connected* zero level-set.
- Near *connections*, $|\psi_S|^2 > \sum_{m\in S}(T_m g)^2$.
- There are several **closed-form distances** between oriented point-sets induced by ψ.

Setup for RDM

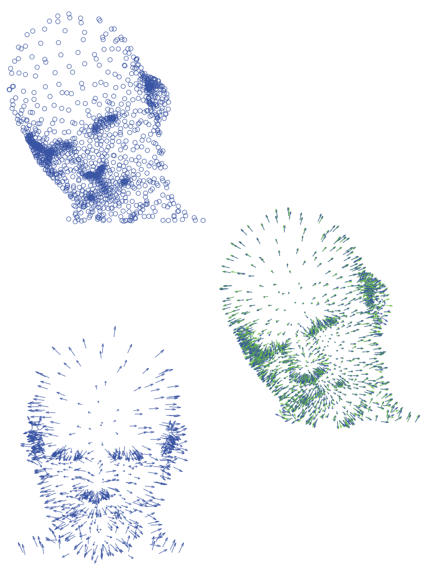

Figure 2: Give a template of oriented template points and a target of unoriented target points of different pose or detail, we aim to obtain a registration of the template onto the target while also reconstructing the normal vectors underlying the target data.

Registering Shapes

We minimize

$$D(S, \phi\cdot T) = ||\psi_S - \psi_{\phi\cdot T}||^2 + \beta\mathcal{R}(\phi) \quad (2)$$

over ϕ represented by parameters for a spline.
ϕ acts on the oriented point-set $T = \{m_a, \nu_a\}_{a=1}^N$ by

$$\phi\cdot T = \{\phi(m_a), (J\phi)_{m_a}\nu_a\}_{a=1}^N.$$

The total derivative wrt $\mathbf{C}$ is

$$\nabla_{\mathbf{C}} I_{(q,\omega)}^{\phi_{\mathbf{C}}\cdot(m,\nu)} = \frac{dI_{(q,\omega)}^{\phi_{\mathbf{C}}\cdot(m,\nu)}}{d\phi_{\mathbf{C}}(m)}\frac{d\phi_{\mathbf{C}}(m)}{d\mathbf{C}} + \frac{dI_{(q,\omega)}^{\phi_{\mathbf{C}}\cdot(m,\nu)}}{d\phi_{\mathbf{C}}\cdot\nu}\frac{d[\phi_{\mathbf{C}}\cdot\nu]}{d\mathbf{C}}.$$

$$\frac{d[\phi_{\mathbf{C}}(m_j)]^{(a)}}{d\mathbf{C}} = \mathbf{R}_j e_a, \quad \frac{d\left[\phi'_{\mathbf{C}}|_{m_j}\nu_j\right]^{(a)}}{d\mathbf{C}} = ([\mathbf{R}'_j]^{(a)})\nu_j^T$$

where $\mathbf{R}$ is the Gram matrix for the spline, e_a is the a^{th} basis vector, and $[\mathbf{R}'_j]^{(a)}$ is the derivative in the a^{th} coordinate direction at control point j.With the TPS or GRBF

$$\mathcal{R}(\phi) = \text{tr}(C^T RC). \quad (3)$$

In the paper, we used a standard L-BFGS solver provided with Matlab®'s optimization package, and used TPS.

Takeaway

We present a closed-form registration model that matches implicit curves given by sparse oriented points. One curve can be estimated as the registration is obtained.

Registration Results

We compared the registration performance of RDM against other point-based (and field-based) algorithms on the Gatobait dataset.

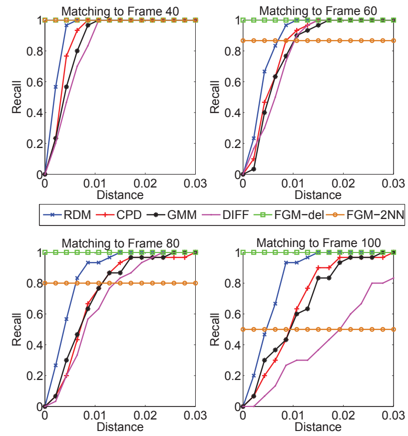

Figure 3: Precision/Recall curves for the CMU-House dataset. ECCV paper also includes evaluation of RDM on 3d datasets.

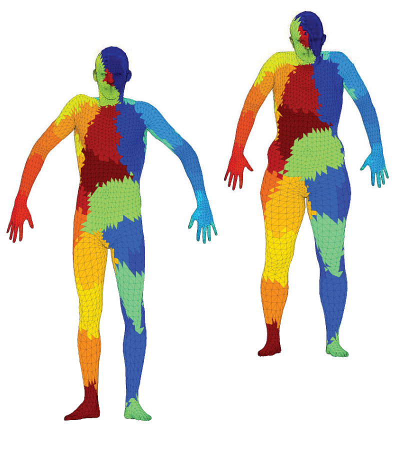

Figure 4: Voronoi regions for a subset of correspondences obtained from RDM registration between two different subjects in similar poses.

Highlights

- ψ is a linear representation providing approximate signed distance in near field.
- Connects density-based shape approach with level-set approach, allowing to sample points or recover closed curves.
- A method for reconstructing while registering by combining density and geometry in 1 field.
- RDM shows that for certain tasks *sparsely sampled shape information* is enough to provide an improvement over point-based methods.

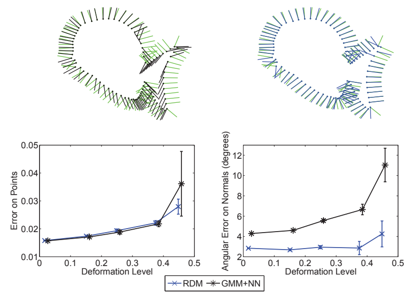

Figure 5: A comparison of normal recovery approaches based on transferring the normal vector from a given registration vs. RDM. Simultaneously estimating the normal vector while solving the transforming problem leads to more reliable transformations and more accurate normal estimates.

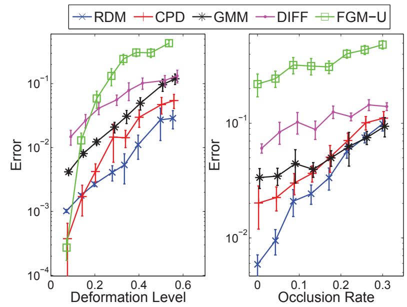

Figure 6: RDM outperforms standard methods when considering multi-curve datasets. See the paper for a comparison of performance out-of-sample.

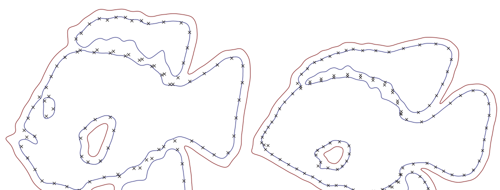

Figure 7: Examples of reconstructed curves using RDM.

Directions for Further Work

- Study different metrics for (2), anisotropic kernels.
- Organize both the template and the target.
- Alternate motion models.
- Improve the optimization strategy.
- Implicit representations of shape handle connection changes easily. Is there a spline or deformation model suited to the problem of registering different topologies?

Code and Dataset Available

`https://github.com/johncorring/RDM`

- Web: `http://www.cise.ufl.edu/~corring`
- Email: johncorring@gmail.com