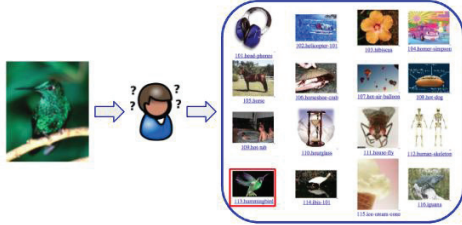


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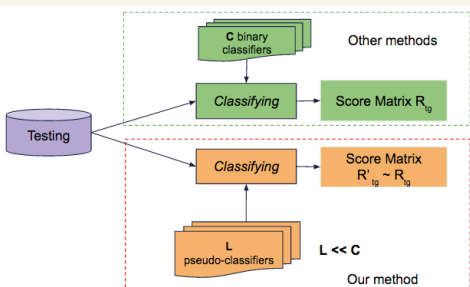
Introduction



- **Task:** Multi-class Classification → Given an image, predict which class that it belongs to.
- **Goal:** Reducing the test time complexity → necessary when the number of classes is large.

- **Challenges:**
 - The complexity in test time grows linearly with the number of classes when using the standard OvA approach.
 - The label tree approach usually suffers the well-known error propagation problem and it is difficult for parallelization for further speedup.

- **Proposal:**
 - The key idea is to use a smaller number of classifier evaluations to reduce the testing cost.
 - It is similar to the label tree approach, but hierarchical structure is not used.
 - It is done through prediction score matrix decomposition.



Method Overview

Let's revisit SVD.

$$R = U \Sigma V^T \quad (2)$$

$$RR^T = U \Sigma V^T V \Sigma U^T = U \Sigma^2 U^T \quad (3)$$

$$RR^T U = U \Sigma^2 \quad (4)$$

Instead of obtaining U directly by singular value decomposition, we take into account that U is the result of performing regression on the feature vectors of the images. To do so, we will pose the original problem as an eigenvalue problem,

$$RR^T u = \lambda u \quad (5)$$

$$u^T RR^T u = u^T \lambda u = \lambda \quad (6)$$

where u is an eigenvector and λ is the corresponding eigenvalue.

Now we consider u as regression result, namely, $u_{(i,k)} \approx g_k(v_i)$. We further assume linear regression:

$$g_k(v_i) = \langle w_k, v_i \rangle + b_k \quad (7)$$

$$= [w_k \ b_k] [v_i \ 1]^T \stackrel{\text{def}}{=} \tilde{w}_k^T \tilde{v}_i \quad (8)$$

By defining the matrix of features:

$$\tilde{S} = \begin{bmatrix} \tilde{v}_1 & \tilde{v}_2 & \dots & \tilde{v}_N \\ 1 & 1 & \dots & 1 \end{bmatrix} \quad (9)$$

we want $u \approx \tilde{S}^T \tilde{w}$. Substituting this into (6), the problem becomes:

$$\text{maximize } \tilde{w}^T \tilde{S} RR^T \tilde{S}^T \tilde{w} \quad (10)$$

$$\text{such that } \tilde{w}^T \tilde{S} \tilde{S}^T \tilde{w} = 1 \quad (11)$$

We can use the Lagrange multipliers method to solve it.

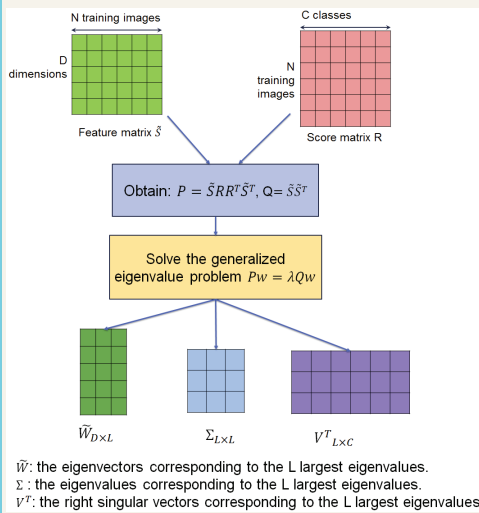
$$J = \tilde{w}^T \tilde{S} RR^T \tilde{S}^T \tilde{w} - \lambda (\tilde{w}^T \tilde{S} \tilde{S}^T \tilde{w} - 1) \quad (12)$$

$$\frac{\partial J}{\partial \tilde{w}} = 2 \tilde{S} RR^T \tilde{S}^T \tilde{w} - 2 \lambda \tilde{S} \tilde{S}^T \tilde{w} = 0 \quad (13)$$

$$\tilde{S} RR^T \tilde{S}^T \tilde{w} = \lambda \tilde{S} \tilde{S}^T \tilde{w} \quad (14)$$

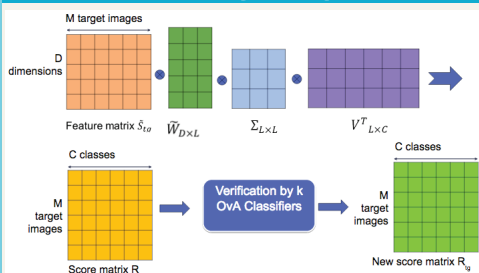
The above can be regarded as a generalized eigenvalue problem $Pw = \lambda Qw$, where $P = \tilde{S} RR^T \tilde{S}^T$ and $Q = \tilde{S} \tilde{S}^T$.

Training Stage



\tilde{W} : the eigenvectors corresponding to the L largest eigenvalues.
 Σ : the eigenvalues corresponding to the L largest eigenvalues.
 \tilde{V}^T : the right singular vectors corresponding to the L largest eigenvalues.

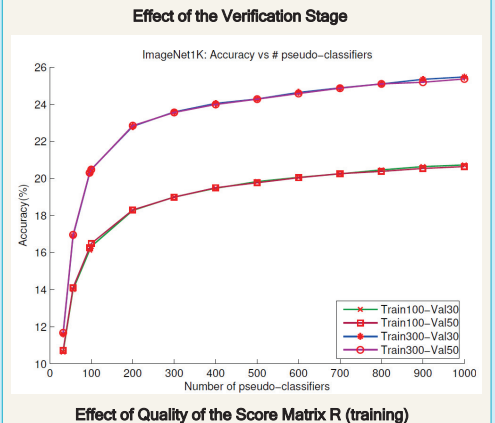
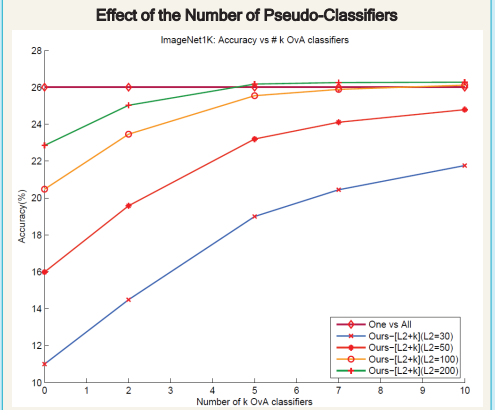
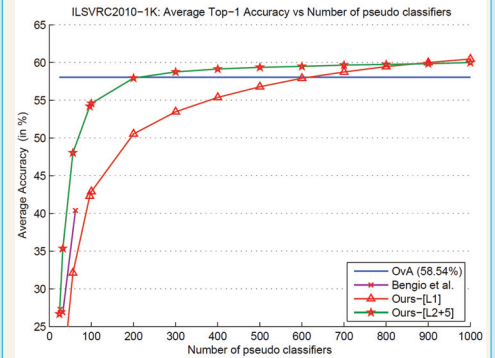
Testing Stage



Accuracy and Test Speedup

Method	Flat	$T_{32.2}$ (L=96)	$T_{10.2}$ (L=56)	$T_{4.4}$ (L=32)
	Acc(%)	Acc(%)	Acc(%)	Acc(%)
Fast-Balanced Tree [10]	11.9	10.3	8.92	18.20
Probabilistic Tree [24]	21.38	10.42	20.54	17.85
RDOC [2]	3.35	10.42	2.15	17.86
RSOC [2]	3.48	10.42	2.18	17.86
Spectral ECOC [39]	7.18	10.42	5.57	17.86
Attribute-based learning [21]	12.13	10.42	8.07	17.86
Ours-[L]	20.31	10.42	16.96	17.86
Ours-[L ₂ + k]	25.38	10.42	23.32	17.86
1K-OvA(LIBLINEAR)	26.01	1.0		

Results



Effect of Quality of the Score Matrix R (training)

Conclusion

- Propose an efficient method for large scale image classification.
- The method is based on joint optimization for prediction score matrix decomposition.
- Comprehensive evaluations on large datasets such as ImageNet-1K, ImageNet-10K show superiority over SoA methods such as OvA and label tree methods.