Pseudo-Geometric Formulation for Fitting Equidistant Parallel Lines

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Objective:

A method that is able to rapidly and robustly determine the projective spacing model from a minimum set of assigned line combinations.

Prior Art:

Let a point in homogeneous form be **p**=(x,y,z) and line **l**=(a,b,c). A group of equally space parallel lines can be represented as $ax + by + \lambda = 0$, where $\lambda = 0,1,\ldots,n$ (is a scalar index of a line) takes only integer values. Under perspective imaging the line transformation is [1]

$$\mathbf{l}_\lambda = \mathbf{H}^{-\mathrm{T}}\begin{bmatrix} a \\ b \\ \lambda \end{bmatrix} = \mathbf{H}^{-\mathrm{T}}\begin{bmatrix} a & 0 \\ b & 0 \\ \lambda & 1 \end{bmatrix}\begin{bmatrix} 1 \\ \lambda \end{bmatrix} = \mathbf{A}\begin{bmatrix} 1 \\ \lambda \end{bmatrix} \quad (1)$$

where $\mathbf{H}$ is the homography matrix and $\mathbf{A} = [\mathbf{l}_0 \;\; \mathbf{l}_\infty]$ is a 3 by 2 matrix. This leads to vector cross product [2] as follows

$$\mathbf{l}_\lambda \times \mathbf{A}\begin{bmatrix} 1 \\ \lambda \end{bmatrix} = 0 \quad (2)$$

which can be arranged in $\mathbf{Z}_b\mathbf{x}_b = 0$ format and solved using Singular Value Decomposition (*SVD*) for $\mathbf{x}_b$, i.e., $\mathbf{l}_0$ and $\mathbf{l}_\infty$ [2].

Pseudo-Geometric Formulation:

We know from Eqn. 1 that

$$\mathbf{l}_\lambda = \mathbf{l}_0 + \lambda\mathbf{l}_\infty \quad (3)$$

First, we rework this formulation to interpolate between real lines avoiding the use of line at infinity $\mathbf{l}_\infty$. For line n we get

$$\mathbf{l}_n = \mathbf{l}_0 + n\mathbf{l}_\infty \quad (4)$$

Re-arranging gives us

$$\mathbf{l}_\infty = {(\mathbf{l}_n - \mathbf{l}_0)}/{n} \quad (5)$$

Substituting Eqn. 5 in Eqn. 3 to eliminate $\mathbf{l}_\infty$ to get

$$\mathbf{l}_\lambda = \frac{(n-\lambda)\mathbf{l}_0 + \lambda\mathbf{l}_n}{n} \quad (6)$$

Second, we exploit the dot product between the interpolated lines of the equidistant projective spacing model and points on edge lines in the image, i.e.,

$$\mathbf{l}_\lambda \cdot \mathbf{p} = \mathbf{0}$$

$$\frac{(n-\lambda)\mathbf{l}_0 + \lambda\mathbf{l}_n}{n} \cdot (x, y, 1) = 0 \quad (7)$$

$$(n-\ \lambda)a_0x + (n-\ \lambda)b_0y + (n-\ \lambda)c_0 + \lambda a_n + \lambda b_n + \lambda c_n = 0$$

and can be arranged in $\mathbf{Zx} = 0$ format as

$$\begin{bmatrix} (n-\ \lambda_1)x_{1,1} & (n-\lambda_1)y_{1,1} & (n-\ \lambda_1) & x_{1,1}\lambda_1 & y_{1,1}\lambda_1 & \lambda_1 \\ (n-\ \lambda_1)x_{1,2} & (n-\lambda_1)y_{1,2} & (n-\ \lambda_1) & x_{1,2}\lambda_1 & y_{1,2}\lambda_1 & \lambda_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (n-\ \lambda_i)x_{i,j} & (n-\ \lambda_i)y_{i,j} & (n-\ \lambda_i) & x_{i,j}\lambda_i & y_{i,j}\lambda_i & \lambda_i \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (n-\ \lambda_N)x_{N,M} & (n-\ \lambda_N)y_{N,M} & (n-\ \lambda_N) & x_{N,M}\lambda_N & x_{N,M}\lambda_N & \lambda_N \end{bmatrix}\begin{bmatrix} a_0 \\ b_0 \\ c_0 \\ a_n \\ b_n \\ c_n \end{bmatrix} = 0 \quad (8)$$

and solved using *SVD* for $\mathbf{x}$, i.e., $\mathbf{l}_0 = (a_0, b_0, c_0)$ and $\mathbf{l}_\infty = (a_n, b_n, c_n)$. Here, λ_i, $i = 1,2,\ldots,N$, is the index of a line represented by *M* points.

Experimental Results:

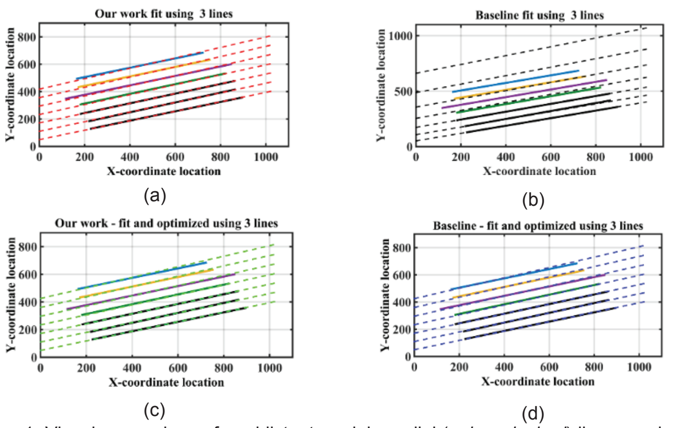

Figure 1. Visual comparison of equidistant model parallel (*colour dashed*) lines overlayed on all the image (*bold colour and black*) lines: (a) our work, (b) baseline, (c) our work optimized, and (d) baseline optimized, when fitted using three image lines (*black lines*).

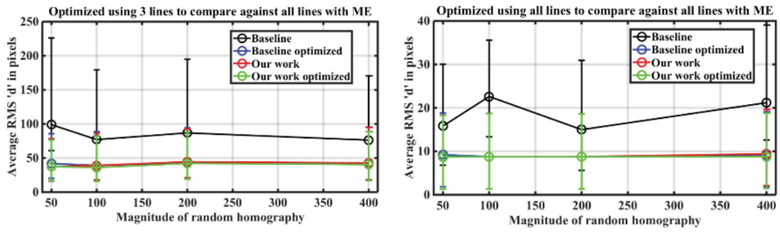

Figure 2. Average root mean square *RMS* perpendicular distance *d* in pixels with (25%-75% percentiles) error bars. Fitting using three (*left column*) and all (*right column*) lines with measurement error (*ME*) of 8 pixels.

Table 1: Average *RMS d* in pixels and standard deviation across homographies measured against all lines with imaging error (ME), when three and all lines with ME are used for fitting.

	Fitted using 3 lines with imaging error			
	2	4	8	16
Our work	**7.27** ± 0.76	19.8 ± 8.02	**40.9** ± 3.14	**95.3** ± 10.8
Baseline	22.9 ± 4.11	48.5 ± 8.45	84.8 ± 10.6	157.2 ± 11
Our Infinity	7.3 ± 0.8	**19.6** ± 7.9	44.3 ± 5.5	99.3 ± 10.7
Our work optimized	**7.2** ± 0.7	19.3 ± 7.7	39.1 ± 2.7	86 ± 7.4
Baseline optimized	7.92 ± 0.8	22.6 ± 9.3	41.7 ± 2.11	89.4 ± 7.7
Our Infinity optimized	7.2 ± 0.6	**19.1** ± 7.3	**39.01** ± 2.8	**83.2** ± 7.6

	Fitted using all lines with imaging error			
	2	4	8	16
Our work	**2.17** ± 0.03	**4.49** ± 0.16	**8.9** ± 0.33	**18.26** ± 0.79
Baseline	3.34 ± 0.38	7.53 ± 1.37	18.6 ± 3.8	40.1 ± 5.3
Our Infinity	2.17 ± 0.03	4.6 ± 0.2	9.4 ± 1.2	19.2 ± 1.4
Our work optimized	**2.16** ± 0.03	**4.43** ± 0.18	**8.75** ± 0.04	**17.6** ± 0.49
Baseline optimized	2.16 ± 0.03	4.44 ± 0.18	8.9 ± 0.21	17.8 ± 0.38
Our Infinity optimized	2.16 ± 0.03	4.43 ± 0.18	8.9 ± 0.34	17.6 ± 0.47

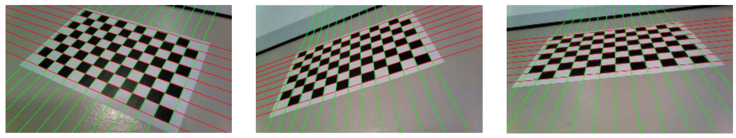

Figure 3. Equidistant model parallel *(Red and Green colour) lines overlayed* on the perspectively distorted checkerboard grid images.

Table 2: Average *RMS d* in pixels with one standard deviation across 30 images of checkerboard grid images.

	First set of equidistant parallel lines			
	Fit 3 lines Conditioned	Fit 3 lines Original	Fit all lines Conditioned	Fit 3 lines Original
Our work	**8.45** ± 2.7	**8.44** ± 2.71	**1.5** ± 0.43	**1.51** ± 0.43
Baseline	10.43 ± 4.53	23.03 ± 29.18	1.7 ± 0.46	2 ± 1.3
Our Infinity	8.45 ± 2.72	8.46 ± 2.72	1.51 ± 0.43	1.51 ± 0.43
Our work optimized	**8.44** ± 2.71	**8.44** ± 2.71	**1.5** ± 0.43	**1.51** ± 0.43
Baseline optimized	8.71 ± 2.81	21.16 ± 23.36	1.5 ± 0.43	1.9 ± 1
Our Infinity optimized	8.45 ± 2.72	8.46 ± 2.72	1.5 ± 0.43	1.51 ± 0.43

	Second set of equidistant parallel lines			
	Fit 3 lines Conditioned	Fit 3 lines Original	Fit all lines Conditioned	Fit 3 lines Original
Our work	**5.42** ± 1.8	**5.42** ± 1.8	**1.1** ± 0.5	**1.1** ± 0.5
Baseline	7.3 ± 4.7	18.75 ± 53.1	1.6 ± 1.33	1.55 ± 1.41
Our Infinity	5.42 ± 1.81	5.71 ± 2.6	1.1 ± 0.5	1.1 ± 0.5
Our work optimized	**5.42** ± 1.8	**5.42** ± 1.8	**1.1** ± 0.5	**1.1** ± 0.5
Baseline optimized	5.65 ± 2	20.5 ± 63.4	1.1 ± 0.5	1.33 ± 0.7
Our Infinity optimized	5.42 ± 1.81	5.71 ± 2.6	1.1 ± 0.5	1.1 ± 0.5

Figure 4. Equidistant parallel (*coloured*) lines detected on York data set [3].

Table 3: Average computational time in seconds with one standard deviation across 1600 images of the simulated dataset.

	Linear	Non-Linear optimized
Our work	**0.0002** ± 0.0003	**0.0018** ± 0.0023
Baseline	0.0002 ± 0.0003	0.104 ± 0.0234
Our Infinity	0.0002 ± 0.0002	0.0853 ± 0.0281

Conclusion:

We proposed a linear pseudo-geometric formulation which is based on the dot product of interpolated model lines against end points in the image for equidistant parallel line fitting under perspective distortion. Simulated and real data experiments showed that our improved solution does not require any pre-conditioning of the image data and avoids the need for computationally expensive non-linear optimization.

References:

1. **Schaffalitzky, F., Zisserman, A.**: Planar grouping for automatic detection of vanishing lines and points. Image and Vision Computing **18(9) (2000) 647-658.**

2. **Hartley, R., Zisserman, A.:** Multiple View Geometry in Computer Vision. 2 edn. Cambridge University Press **(2003)**.

3. **Denis, P., Elder, J., Estrada, F.:** Efficient edge-based methods for estimating manhattan frames in urban imagery. In: 10th European Conference on Computer Vision, Proceedings, Part II, Springer Berlin Heidelberg (**Oct 2008**) **197-210**.