A Generalized Successive Shortest Path Solver for Tracking Dividing Targets

Carsten Haubold, Janez Aleš, Steffen Wolf, Fred A. Hamprecht
HCI/IWR, University of Heidelberg

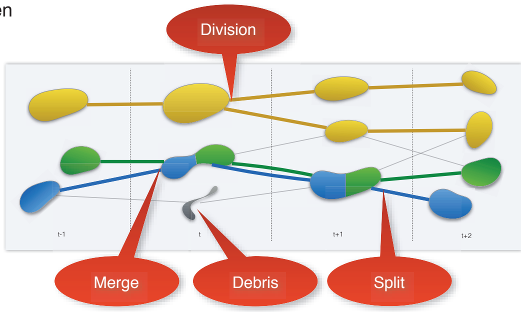
Universität Heidelberg Zukunft seit 1386 · IWR Interdisciplinary Center for Scientific Computing · Heidelberg Collaboratory for Image Processing · HGS MathComp · SFB 1129

Introduction

- tracking-by-assignment is an important model for multi-target tracking
- often solved optimally by min-cost flow algorithms [Zhang, Pirsiavash, Lenz]
- when allowing for dividing targets (as in cell tracking), problem is NP-hard
- can be addressed with integer linear programming (ILP), but that does not scale
- we propose a network-flow based approximation with compelling anytime performance

Network Flow Setting

- we build a global model to incorporate temporal context akin to [Schiegg, Magnusson]
- assume imperfect segmentation: targets can merge and split, and spurious detections can happen



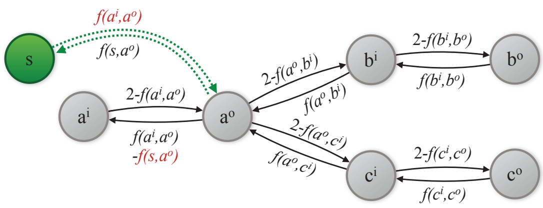
- set up a directed graph $G=(V,E)$ as [Zhang], every unit of flow tracks one target
  - detections represented as two nodes, connecting arc holds the detection cost
- arc costs $\forall_{(u,v)\in E}\ w(u,v,f(u,v)) \in \mathbb{R}$, where $f(u,v)$ is the current flow along (u,v). w(u,v,f(u,v)) must be convex w.r.t. f(u,v)
- arc capacities c(e) bounded by the maximally allowed mergers

Generalized Successive Shortest Paths (SSP)

- residual graph arc capacities defined as $c_r(u,v) = c(u,v) - f(u,v)$, $c_r(v,u) = f(u,v)$
- **Observation:** residual arc capacity depends on the flow along the arc itself!

→ handle constraints by adjusting residual arc capacities between SSP iterations!

- allow residual arc capacities to depend on the **flow along other** arcs, e.g. $c_r(u,v) = f(\bar{u}, v)$



Conditioned Residual Capacity Successive Shortest Paths (crcSSP) Algorithm:

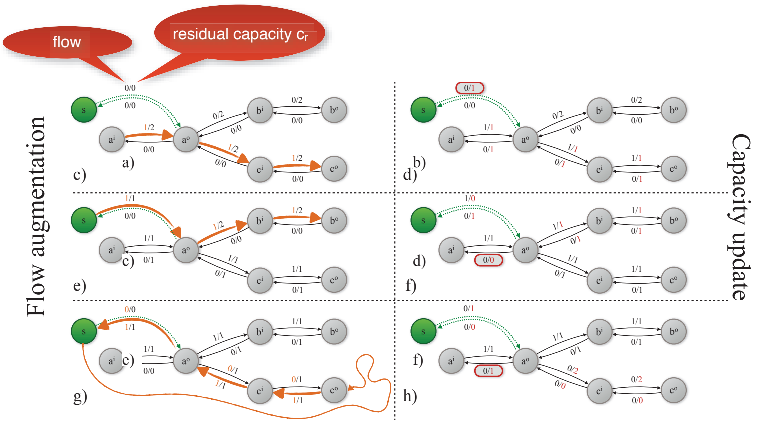
- augment SSP by additional capacity update rule
- falls back to normal SSP when **no constraints** (e.g. divisions) are present → **optimal!**
- add paths as long as they have negative cost, otherwise we would leave a minimum
- depending on the order of found paths and the **capacity changes incurred by constraints**, the optimal solution might be blocked → **greedy!**

Residual Graph Approximations:

- [Magnusson]'s *"swap arc"* heuristic can now be understood as a second order residual graph approximation, where paths can only go backward for one frame at a time
- a first order approximation would mean no arcs pointing backward in time
- both can be solved by dynamic programming

Implementation Details:

- graph contains negative cost cycles → use Bellman Ford (BF) to find shortest paths
- improve BF runtime:
  - add early stopping criteria to notice when a cycle is present:
    → check for cycle when source distance decreases or path longer than average
  - order edges by time before processing them with BF (`-o`)
  - reuse all computed distances that are still valid from the previous iteration (`-i`)



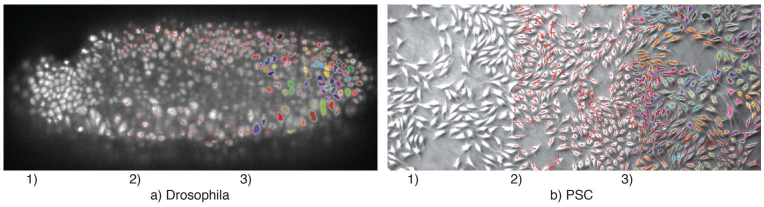
Experiments & Results

Datasets: (both with given ground truth)

a) *Drosophila* (from [Schiegg]), 100 time frames, 45k nodes, 110k arcs, 10k division candidates

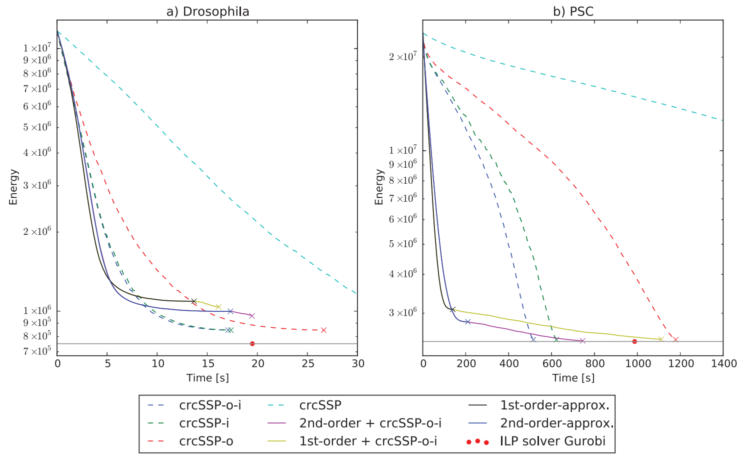
b) Pancreatic Rat Stem Cells (*PSC*) (from [Rapoport]), 104 time frames, 260k nodes and 770k arcs, 126k division candidates

Exemplary 2D slices of the 3D+t and 2D+t datasets, each showing from left to right:
1) raw data
2) raw data with resulting tracks, 10 frames before and after the current frame
3) our segmentation as additional overlay, where color indicates shared ancestry.



Anytime Performance:

- energy of solution after every new path using different SSP settings, compared to the optimal ILP energy
  - 1st and 2nd order approximations are faster but lead to a larger optimality gap
  - ordering arcs and only recomputing invalidated distances in BF is crucial
  - warm-starting `crcSSP` from 1st or 2nd order approximations performs poorly
- `crcSSP` outperforms the ILP solver in runtime while using at most half the RAM
  - is scalable: runtime complexity $O(P * N^2)$, where P=#tracks, N=#nodes in graph



Tracking Quality:

- comparing the tracking results with the ground truth shows that, despite the heuristic nature our flow-based solver, the results are very close to the optimal ILP solution

		1st order	2nd order	ours	ILP (Gurobi)
a) Drosophila	Quality (F-Score)	0.89	0.90	0.92	0.94
	Runtime	13s	17s	17s	19s
b) PSC	Quality (F-Score)	0.78	0.82	0.91	0.91
	Runtime	140s	210s	515s	987s

Conclusion & Future Work

- we presented a new approximate primal feasible solver to handle side-constraints in network flow problems, such as division constraints in cell tracking
- in practice it gives close-to-optimal results, while providing iteratively improving intermediate solutions (useful e.g. for displaying interactive feedback)
- we want to apply this to other side-constraints in tracking

Source Code available at GitHub:

- generalized SSP: `https://github.com/chaubold/dpct`
- ILP solver: `https://github.com/chaubold/multiHypothesesTracking`

References

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