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Patch-Based Low-Rank Matrix Completion for Learning of Shape and Motion Models from Few Training Samples

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1 Motivation

- The **high-dimension-low-sample-size problem** limits the flexibility of **statistical shape models** (SSMs).
- Collecting an adequately large and representative training population is often laborious and challenging, particularly in medical applications.
- Our approach: **assumption of locality**, i.e. we assume that local variations in shape, intensity or motion have limited effects in distant areas.
- Our method allows the model to **combine local variations observed in different training samples** while preserving overall object properties, i.e. generating valid instances.

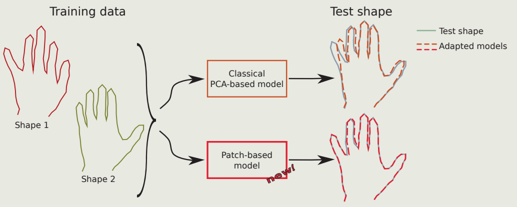

Fig. 1: Example application of the patch-based modeling approach using only two training shapes: Classical models only learn the global transition between the two shapes. The patch-based model combines local shape details, and can adapt to test shapes showing local properties of both shapes.

2 Methods

Patch-based Sample Generation

- **Random patches** of different training shapes are combined in a **virtual sample**.
- Many virtual samples form the **sparse data matrix** $\mathbf{X} \in \mathbb{R}^{m \times n}$.
- Patch sizes and patch distances depend on the desired level of locality.

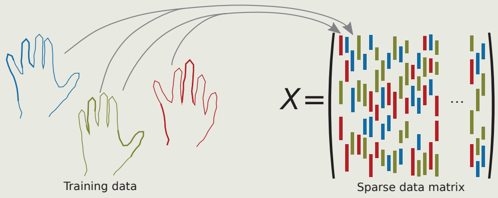

Fig. 2: Virtual sample generation by combining random patches of different shapes in each matrix column.

Low-Rank Matrix Completion

- Solve the matrix-completion problem:

$$\hat{\mathbf{M}} = \arg\min_{\mathbf{M}} \|\mathcal{P}_\Omega(\mathbf{X}) - \mathcal{P}_\Omega(\mathbf{M})\|_F^2 \quad \text{s.t. } rank(\mathbf{M}) = k, \qquad (1)$$

with projection operator $(\mathcal{P}_\Omega(\mathbf{X}))_{ij} = \begin{cases} \mathbf{X}_{ij} & (i,j) \in \Omega \\ 0 & \text{else} \end{cases}$.

- We use **polar incremental matrix completion (PIMC)** [1] to solve Eq. (1):
  - online algorithm based on GROUSE [2]
  - suitable for highly ill-conditioned data matrices $\mathbf{X}$

Patch-based Model

- Solution of Eq. (1) is the matrix $\mathbf{M} = \mathbf{U}\mathbf{R}^T$ of given rank k.
- Model basis $\mathbf{U}$ is applied to approximate a new sample $\boldsymbol{y}$ by

$$\hat{\boldsymbol{y}} = \mathbf{U}\hat{\boldsymbol{w}} \quad \text{with} \quad \hat{\boldsymbol{w}} = \arg\min_{\boldsymbol{w}} \|\mathbf{U}\boldsymbol{w} - \boldsymbol{y}\|_2^2, \qquad (2)$$

- Distributions of $\boldsymbol{w} \sim \mathcal{N}(\boldsymbol{\mu}, diag(\sigma_1, \ldots, \sigma_k))$ are estimated from $\mathbf{R}$.

3 Experiments

- **Evaluation methods:** Computing *generalization errors* and *specificity errors* for varying numbers of available training samples.
- Performance is demonstrated in three different **applications:**
  - **IMM face data:** 2D contour of 58 facial landmarks ($m = 116$) (see Fig. 3).
  - **LIDC lung data:** 3D surfaces with 2000 pseudo-landmarks ($m = 6000$)
  - **Respiratory lung motion:** 2D motion fields ($m = 64000$) (see Fig. 4).

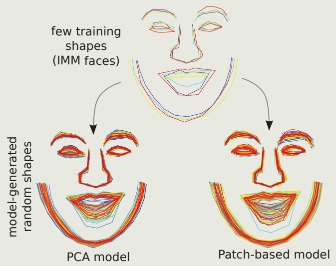

Fig. 3: IMM faces: Model-generated random contours (bottom) from N=4 training samples (top). The patch-based model shows a higher variability while preserving the overall shape.

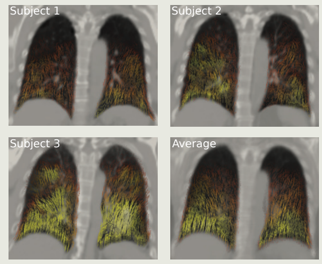

Fig. 4: Respiratory lung motion: Motion fields describing the lung deformation of three example subjects and the average motion of the generated model.

4 Results

- A *ROC-like analysis* shows that our model clearly outperforms classical PCA and FEM-PCA models [3].
- In all applications the generalization error is improved for small training sizes (by ≈20% - 40%).
- As expected, improvements in terms of generalization ability come along with slightly increased specificity errors.

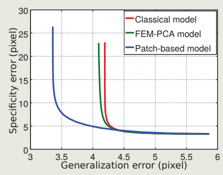

Fig. 5: *ROC-like analysis* for N = 10 samples of IMM faces.

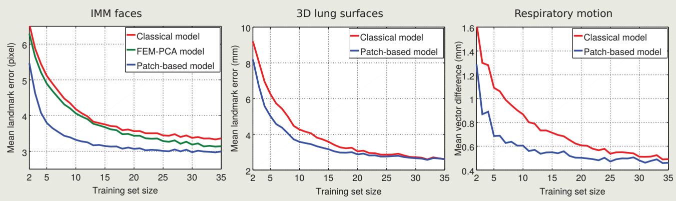

Fig. 6: Generalization errors for three experiments comparing the patch-based model (blue), the classical model (red), and the FEM-PCA model [3] (green, IMM faces only) given a varying numbers of training samples. Smaller values indicate better models.

5 Conclusion

- The proposed method can be applied for a variety of problems.
- Leads to an increased flexibility and generalization ability while the validity of generated model instances is preserved.
- Online ability of the PIMC algorithm avoids explicit storage of data matrix $\mathbf{X}$ in large scale problems, e.g. deformation modeling.
- Although only local information was provided, the proposed method is able to learn global shape variability as well (see Fig. 7).

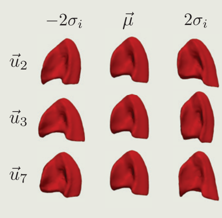

Fig. 7: 3D lung surfaces: Mean surface and three associated deformation modes of the generated patch-based model.

[1] Kennedy R, Taylor CJ, Balzano L. Online Completion of Ill-conditioned Low-Rank Matrices. In: IEEE Global Conference on Signal and Information Processing (GlobalSIP); 2014. p. 507–511.
[2] Balzano L, Nowak R, Recht B. Online identification and tracking of subspaces from highly incomplete information. In: Communication, Control, and Computing (Allerton); 2010. p. 704–711.
[3] Cootes TF, Taylor CJ. Combining point distribution models with shape models based on finite element analysis. Image and Vision Computing. 1995;13(5):403–409.

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