GLOBAL REGISTRATION OF 3D POINT SETS VIA LRS DECOMPOSITION

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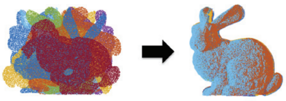
1. Introduction

Multiview Registration

The goal of multiview registration is to find the **rigid transformations** $M_i \in SE(3)$ that bring multiple ($n \geq 2$) 3D point sets into alignment.



We focus on **global** methods which consider simultaneously all the point sets $\Longrightarrow$ no drift

Our Contribution

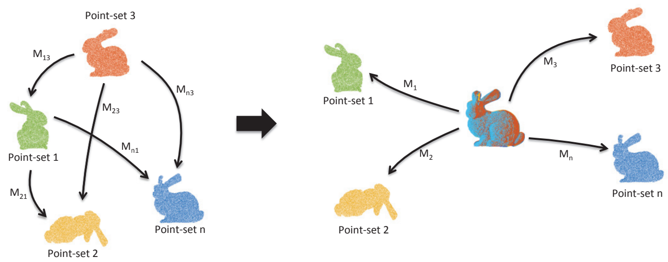
We show that the registration problem can be expressed as a **low-rank and sparse** (LRS) matrix decomposition $\Longrightarrow$ **robustness** to outliers

2. Problem Definition

Multiview registration can be formulated as a **motion averaging/synchronization** problem *without* involving 3D points

$$M_{ij} = M_i M_j^{-1} \quad \forall (i,j) \in \mathcal{E}$$

where $M_{ij} \in SE(3)$ is the rigid transformation that aligns point-set j with point-set i.



Estimates $\widehat{M}_{ij} \in SE(3)$ are computed through the **Iterative Closest Point** (ICP) algorithm.

3. Proposed Approach

Notation

The synchronization constraint can be expressed as $X = MM^{-\flat} \implies$ **rank**$(X) = 4$

$$X = \begin{pmatrix} I_4 & M_{12} & \dots & M_{1n} \\ M_{21} & I_4 & \dots & M_{2n} \\ \dots & & & \dots \\ M_{n1} & M_{n2} & \dots & I_4 \end{pmatrix}, \quad M = \begin{bmatrix} M_1 \\ M_2 \\ \dots \\ M_n \end{bmatrix}, \quad M^{-\flat} = \left[M_1^{-1} \; M_2^{-1} \; \dots \; M_n^{-1} \right]$$

In the case of *missing data*, the available information is represented by $\mathcal{P}_\Omega(\widehat{X})$, where $\widehat{X}$ denotes a noisy version of X, and $\mathcal{P}_\Omega$ denotes the projection onto the *sampling set* Ω.

Minimization Problem

$$\min_X \left\| \mathcal{P}_\Omega(\widehat{X} - X) \right\|_F^2 \quad \text{s.t. } X = MM^{-\flat}, \; M \in SE(3)^n \quad \Longleftrightarrow \quad \min_{M_i \in SE(3)} \sum_{(i,j)\in\mathcal{E}} \left\| \widehat{M}_{ij} - M_i M_j^{-1} \right\|_F^2$$

additional variable S (outliers)

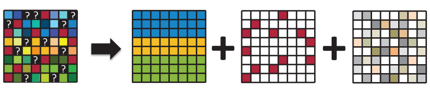
$$\min_{X,S} \left\| \mathcal{P}_\Omega(\widehat{X} - X) - S \right\|_F^2 \quad \text{s.t. } X = MM^{-\flat}, \; M \in SE(3)^n, \; S \text{ sparse in } \Omega$$

rank relaxation $\longrightarrow$

$$\min_{L,S} \left\| \mathcal{P}_\Omega(\widehat{X} - L) - S \right\|_F^2 \quad \text{s.t. rank}(L) \leq 4, \; S \text{ sparse in } \Omega$$

This is a **LRS decomposition**: we aim at recovering the low-rank matrix L starting from an **incomplete** subset of its entries $\mathcal{P}_\Omega(\widehat{X})$ corrupted by **noise** N and **outliers** S.

$$\mathcal{P}_\Omega(\widehat{X}) = \mathcal{P}_\Omega(L) + S + N$$



The solution is then projected onto $SE(3)$ via Singular Value Decomposition (SVD).

4. Low-rank and Sparse Decomposition

R-GoDec

R-GoDec [1] expresses the sparse term as $S = S_1 + S_2$ where S_1 has support on Ω and represents outliers, while S_2 has support on Ω^C and represents completion of missing entries.

$$\min_{L,S_1,S_2} \frac{1}{2} \left\| \mathcal{P}_\Omega(\widehat{X}) - L - S_1 - S_2 \right\|_F^2 + \lambda \|S_1\|_1$$
$$\text{s.t. rank}(L) \leq 4, \; \text{supp}(S_1) \subseteq \Omega, \; \text{supp}(S_2) = \Omega^C$$

GRASTA

$$\min_{S,U,Y} \|S\|_1 \quad \text{s.t. } \mathcal{P}_\Omega(\widehat{X}) = \mathcal{P}_\Omega(UY^T) + S, \; U^T U = I_4$$

L1-ALM

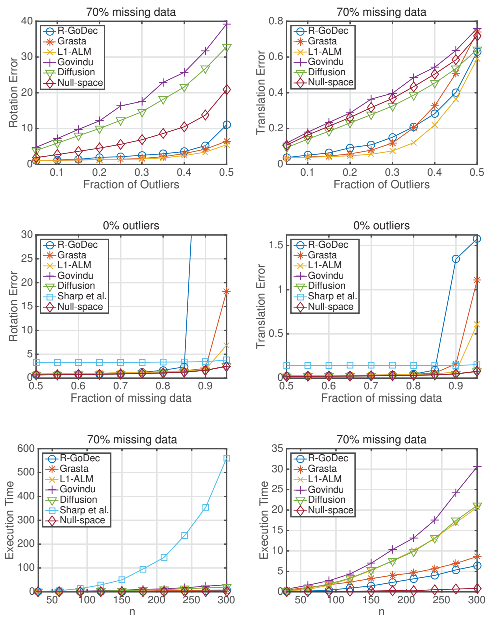
$$\min_{U,Y} \left\| \mathcal{P}_\Omega(\widehat{X} - UY^T) \right\|_1 + \lambda \left\| Y^T \right\|_* \quad \text{s.t. } U^T U = I_4$$

GRASTA [2] and L1-ALM [3] express the LRS decomposition in terms of *subspace identification* where the goal is to identify the column space of the low-rank term.

5. Experiments

We compared our approach with Govindu [4], DIFFUSION [5], Sharp et al. [6] and NULLSPACE [7].

Simulated Data

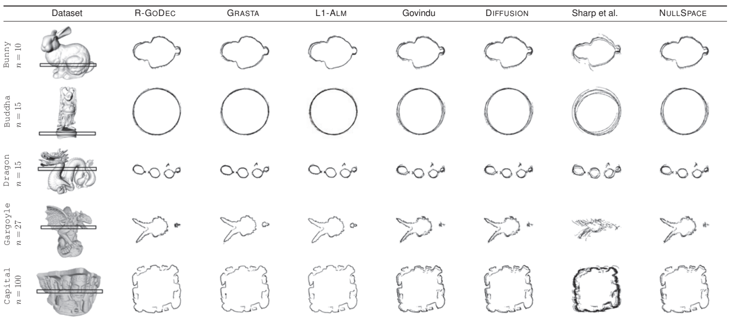


mean errors vs outlier contamination

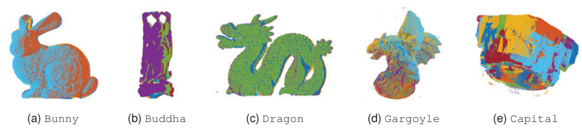
mean errors vs missing data

execution times vs number of point-sets

Real Data



Cross-sections of registered point-sets



3D models obtained with L1-ALM

LRS methods exhibit good performances as for accuracy and execution time, and they outperform all the analyzed techniques in terms of **robustness to outliers**.
However, they are more affected by the sparsity of the graph.

References

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