Structure from Motion on a Sphere

Jonathan Ventura
Department of Computer Science
University of Colorado Colorado Springs, USA

University of Colorado Colorado Springs

INTRODUCTION

- Visual 3D reconstruction requires adequate **baseline** between images.
- However, users of handheld visual SLAM systems typically **rotate** the camera instead of translating it.
- The resulting small translations make structure-from-motion unstable.
- **Idea:** Assume the camera rotates on the surface of a sphere to constrain the structure-from-motion problem.
- For example, with a handheld camera, the shoulder is the origin point and the arm is the fixed radius.
- Possible applications:
- Handheld SLAM initialization
- Stereo panorama creation
- 3D face scan with selfie stick
- Handheld object scanning
- Spherical camera gantry

EPIPOLAR GEOMETRY

Outward-facing camera pose is

$$P_{out} = [R \quad -\mathbf{z}]$$

where

$$z = [0 \quad 0 \quad 1]^T.$$

Relative pose between

$$P_1 = [R_1 \quad -\mathbf{z}] \text{ and } P_2 = [R_2 \quad -\mathbf{z}]$$

is

$$P = [R_2 R_1^T \quad r_3 - \mathbf{z}]$$

where r_3 is the third column of $R_2 R_1^T$.

Essential matrix is

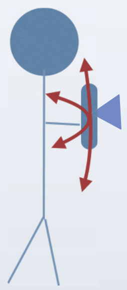
$$E = [r_3 - \mathbf{z}]_\times R_2 R_1^T.$$

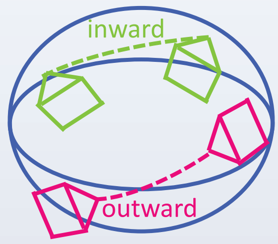
For inward-facing camera, translation is opposite.

Note that camera absolute and relative pose are determined completely by 3-DOF rotations.

CONCEPT

The camera rotates on the surface of a sphere, with its optical axis normal to the surface. The camera could face inward or outward.





THREE-POINT RELATIVE POSE MINIMAL SOLVER

Relative pose is determined by three rotational degrees of freedom
→ need at least three point correspondences

Essential matrix has the form

$$E = \begin{bmatrix} e_1 & e_2 & e_3 \\ e_2 & -e_1 & e_4 \\ e_5 & e_6 & 0 \end{bmatrix}.$$

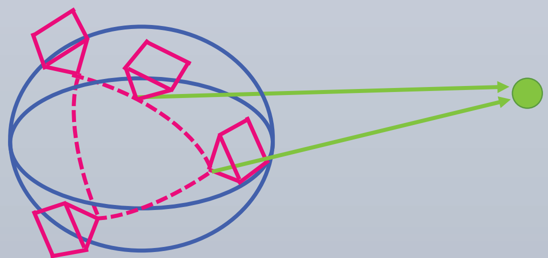
Each correspondence between u_i and v_i gives a constraint of the form

$$v_i^T E u_i = 0.$$

Minimal solver approach:

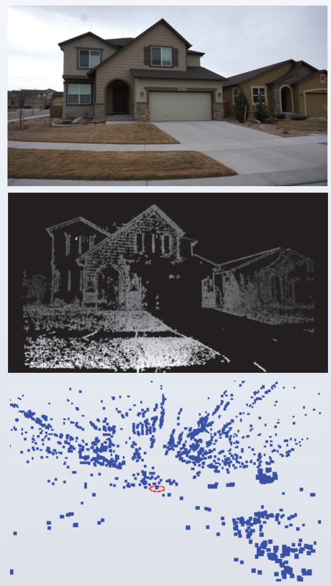
1. Stack linear constraints into an $N{\times}6$ matrix and find $6{\times}3$ nullspace.
2. Apply independent subset of six non-linear constraints from:
$$EE^T E - \frac{1}{2}\text{trace}(EE^T)E = 0$$
3. Solve non-linear constraints using action matrix or hidden-variable resultant.
→ Produces up to four real-valued solutions for E.
4. Select best essential matrix using an extra correspondence.
5. Decompose essential matrix and resolve twisted pair ambiguity using knowledge of inward- or outward-facing cameras.

STRUCTURE FROM MOTION

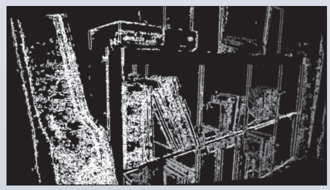


1. Pairwise spherical relative pose estimation
2. Global pose estimation on the sphere via **rotation averaging** [CG13]
3. **Inverse depth bundle adjustment** [YG14] on the sphere
   Camera parameterization: Three rotation parameters
   Point parameterization: Inverse depth in reference frame
   Objective function: Huber cost function on re-projection error

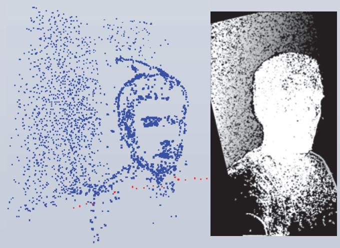
RESULTS



Street scan with handheld camera



Indoor scene -- bookshelf



3D selfie (inward-facing camera)

REFERENCES

[CG13] Chatterjee, A., Madhav Govindu, V.: Efficient and robust large-scale rotation averaging. In: IEEE International Conference on Computer Vision (2013)

[YG14] Yu, F., Gallup, D.: 3D reconstruction from accidental motion. In: 27th IEEE Conference on Computer Vision and Pattern Recognition (2014)

ACKNOWLEDGMENTS

This material is based upon work supported by the National Science Foundation under Grant No. 1464420.