Double-Opponent Vectorial Total Variation

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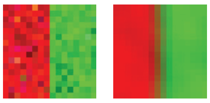
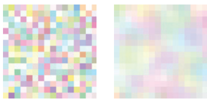
Introduction

We present a new vectorial total variation method that addresses the problem of color consistent image filtering.

- Our approach is inspired from the **double-opponent** cell representation.
- **Existing methods** of vectorial total variation regularizers have **insufficient coupling** between the color channels and thus may introduce color artifacts.
- We **propose** a novel coupling between the color channels via a **pullback-metric** from the opponent space to the data (RGB) space.



Artifact: Artificial colors



Artifact: Color shimmering

Channel mixing

- Modeling psychophysical effects of color in applications is a highly non-trivial problem and many spaces have been proposed, e.g., RGB, sRGB, HSV, YPbPr, YCbCr, CIELAB ...
- We consider the double-opponent color space (see Gao et al. [2013]; Land [1983, 1986]; Ebner [2007]),

$$O = \begin{pmatrix} 1/\sqrt{3} & 0 & 0 \\ 0 & 1/\sqrt{6} & 0 \\ 0 & 0 & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \\ 1 & -1 & 0 \end{pmatrix}$$

(1) Denote by the **linear mapping** $O\colon \mathbb{R}^3 \to \mathbb{R}^3, u = (r,g,b)^\top \mapsto Ou = o = (o_1, o_2, o_3)^\top$, as the transformation from the RGB color space to the double-opponent space.

(2) The **non-linear mapping** to the hue (h), saturation (s) and lightness (L) representation of the opponent space is given by $\psi\colon \mathbb{R}^3 \to \mathbb{R}^3, o \mapsto c = (L,h,s)^\top$ where

$$L = o_1, \quad h = \arctan\begin{pmatrix} o_2 \\ o_3 \end{pmatrix}, \quad s = \left\| \begin{pmatrix} o_2 \\ o_3 \end{pmatrix} \right\|$$

(3) Let $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^3$ denote the **composition** of the linear opponent transform and the mapping $(o_1,o_2,o_3)^\top \mapsto (L,h,s)^\top$ just discussed above, then we define $\varphi\colon u \to \varphi(u) := \psi(Ou)$.

(4) The Euclidean **inner product** $\langle\cdot,\cdot\rangle$ on the Lhs-space induces via φ the pullback metric on the RGB-space (induced by φ) is given by

$$\langle u_1, u_2 \rangle_u := \langle D\varphi(u)u_1, D\varphi(u)u_2 \rangle = \langle u_1, G(u)u_2 \rangle$$
$$G(u) := \big(D\varphi(u)\big)^\top D\varphi(u) = \big(g_{ij}(u)\big)_{i,j=1,2,3}$$

The Jacobian $D\varphi$ and the corresponding metric tensor G, reads

$$D\varphi(u) = \frac{1}{\sqrt{3}}\left(\mathbb{1}, \frac{3}{f}\frac{\alpha}{\|\alpha\|}, \frac{1}{f}\beta\right)^\top$$
$$G(u) = \frac{1}{3}\left(I + \frac{9}{f^4}\alpha\alpha^\top + \frac{1}{f^2}\beta\beta^\top\right)$$

where $\alpha = (b-g, r-b, g-r)^\top$, $\beta = (b+g-2r, b+r-2g, r+g-2b)^\top$, and G has non-normalized eigenvectors $\mathbb{1}, \alpha, \beta$ and corresponding eigenvalues

$$\Lambda = \frac{1}{3} + \operatorname{diag}\left(0, \frac{3}{f^2}, 1\right), \qquad f^2 = f^2(u) = 2(b^2 - bg - br + g^2 - gr + r^2)$$

(5) The channel coupling term is identified from f^2, i.e.,

$$\gamma(u) = f^2 = (b-r)^2 + (r-g)^2 + (g-b)^2 = \|u^\top C\|_F^2, \qquad C = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

Proposed model

- **The energy.** For a **gray-scale image** $u : \Omega \to \mathbb{R}$ defined on a domain $\Omega \subset \mathbb{R}^2$, the total variation measure is

$$TV(u) = \int_\Omega \|\nabla u\| = \sup\left\{ \int_\Omega u \operatorname{div}(\varphi)\, dx \colon \varphi \in \mathcal{C}_c^1(\Omega, \mathbb{R}^2), \|\varphi\|_\infty \le 1 \right\}$$

A function $u \in L^1(\Omega)$ belongs to the space of functions of bounded variation $BV(\Omega)$ if

$$\|u\|_{BV(\Omega)} = \|u\|_{L^1(\Omega)} + TV(u) < \infty.$$

For a **color image** $u : \Omega \to \mathbb{R}^3$ we propose the minimization problem:

$$\min_u \left\{ E(u) = \frac{\mu}{2}\|Ku - g\|^2_{L^2(\Omega)} + \alpha \sum_{i=1}^3 \mathrm{TV}(u_i) + \beta J_{OPP}(u) \right\}$$

the color channel coupling term is

$$J_{OPP}(u) := \int_\Omega \|\nabla Cu\| := \sup_{\|p\|_\infty \le 1} \left\{ \int_\Omega \langle Cu, \operatorname{div}(p) \rangle\, dx \right\}$$

Lemma 1 (Invariance and convexity). *J_{OPP} is rotationally and intensity invariant, 1-homogeneous and convex.*

Lemma 2 (Bounded variation). *Let $u \in BV(\Omega; \mathbb{R}^3)$ then $Cu \in BV(\Omega; \mathbb{R}^3)$.*

Theorem 1 (Uniqueness and existence of solution). *Let $g \in L^\infty(\Omega, \mathbb{R}^3)$ and $u \in BV(\Omega, \mathbb{R}^3)$. Then there exists a unique minimizer u^* of $E(u)$.*

- **Optimization.** We use a Split-Bregman algorithm (Goldstein and Osher [2009]):

$$(u^{k+1}, d^{k+1}, e^{k+1}) = \min_{u,d,e} \frac{\mu}{2}\|Ku - g\|_2^2 + \|d\|_1 + \|e\|_1 + \frac{1}{2}\|B(u,b,d,e)\|_W^2$$

$$b^{k+1} = b^k + \begin{pmatrix} D \\ DC \end{pmatrix} u^{k+1} - \begin{pmatrix} d^{k+1} \\ e^{k+1} \end{pmatrix}, \qquad B(u,b,d,e) := \begin{pmatrix} d \\ e \end{pmatrix} - \begin{pmatrix} D \\ DC \end{pmatrix} u - b$$

and solve

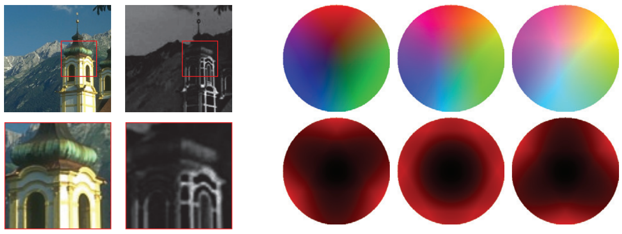
$$u^{k+1} = \min_u \frac{\mu}{2}\|u - g\|_2^2 + \frac{1}{2}\left\|B(u, b^k, d^k, e^k)\right\|_W^2$$

followed by two shrinkage updates for d^{k+1}, e^{k+1}.

Color structure

- Illustration. Color structure in a natural image and isoluminant discs as extracted by γ.

Proposition 1. *γ is invariant to intensity shifts and depends quadratically on color change.*



Results

- **Experimental setup.** The *opponent color components* are corrupted with Gaussian noise.

Original	Noisy (50)	OVTV (ours)	BM3D [Dabov 2007]
PSNR/SSIM/CIEDE	16.3/0.21/21.87	**28.2**/**0.75**/4.01	26.5/0.67/**3.85**

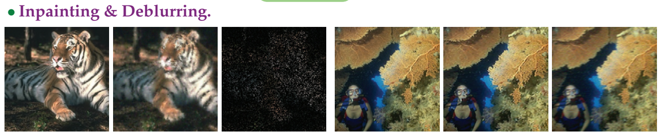
DVTV [Ono '14]	PDVTV [Bresson '08]	TGV [Bredies '10]	BM3DS [Dabov '07]
25.9/0.70/4.19	25.5/0.63/6.62	24.8/0.62/5.52	26.3/0.61/5.60

+ Only our OVTV approach produces **sharp color transitions** and a result similar to the original noise free image.

- **Comparison to other methods.** Error values for (100) Berkeley images,

$\sigma = 20$	DVTV	PDVTV	OVTV	TGV	BM3D	BM3DS
PSNR	29.1 ± 1.94	27.8 ± 1.05	31.6 ± 2.03	26.4 ± 2.43	$\mathbf{32.0} \pm 1.93$	31.9 ± 1.30
SSIM	0.83 ± 0.06	0.82 ± 0.06	$\mathbf{0.89} \pm 0.05$	0.74 ± 0.07	0.88 ± 0.06	0.87 ± 0.05
CIEDE	5.40 ± 1.83	4.69 ± 0.73	2.79 ± 0.65	5.97 ± 3.61	$\mathbf{2.77} \pm 0.54$	3.85 ± 0.64
$\sigma = 50$						
PSNR	24.6 ± 1.85	25.2 ± 1.91	$\mathbf{27.2} \pm 1.94$	24.3 ± 2.07	26.1 ± 2.32	25.9 ± 1.71
SSIM	0.66 ± 0.06	0.72 ± 0.06	$\mathbf{0.78} \pm 0.06$	0.66 ± 0.08	0.72 ± 0.10	0.68 ± 0.09
CIEDE	8.36 ± 1.97	6.10 ± 1.14	5.71 ± 2.17	8.07 ± 2.39	$\mathbf{4.79} \pm 0.85$	6.26 ± 1.01
$\sigma = 80$						
PSNR	21.0 ± 0.96	23.7 ± 1.89	$\mathbf{24.0} \pm 1.73$	22.1 ± 2.00	23.5 ± 2.26	23.4 ± 1.95
SSIM	0.44 ± 0.06	0.69 ± 0.09	$\mathbf{0.69} \pm 0.07$	0.57 ± 0.09	0.63 ± 0.12	0.61 ± 0.10
CIEDE	13.89 ± 1.39	6.76 ± 1.12	8.07 ± 2.11	10.01 ± 2.74	$\mathbf{6.31} \pm 1.08$	7.31 ± 1.03

- **Inpainting & Deblurring.**



Original **Result** **Corrupted** **Original** **Result** **Blurred**

Further results

F. Åström and C. Schnörr "A Geometric Approach to Color Image Regularization", arXiv, http://arxiv.org/abs/1603.05285, 2016