An Efficient Fusion Move Algorithm for the Minimum Cost Lifted Multicut Problem

Thorsten Beier* Björn Andres† Ullrich Köthe* Fred A. Hamprecht*
* HCI/IWR, University of Heidelberg
† Computer Vision and Multimodal Computing, Max Planck Institute for Informatics

Introduction

The Minimum Lifted Multicut Problem:

- An Optimization problem whose feasible solutions are decompositions of a graph
- Objective function can penalize or reward all decompositions for which any given pair of nodes are in distinct components
- We propose a fusion move algorithm which outperforms existing algorithms
- We use this objective function for image segmentation and obtain a new state of the art for a problem in biological image analysis

Multicut Objective [4, 3, 6]

The minimum multicut finds the clustering which minimizes the sum of weights between clusters. Given a graph $G = (V, E)$ with edge weights $c : E \to \mathbb{R}$ this can be formulated as an ILP:

$$\min_{x \in \{0,1\}^E} \sum_{e \in E} c_e x_e \tag{1}$$

$$\text{subject to} \quad \underbrace{\forall Y \in \text{cycles}(G)\ \forall e \in Y :\ x_e \le \sum_{e' \in Y \setminus \{e\}} x_{e'}}_{\text{Ensures valid segmentation without dangling edges}} \tag{2}$$

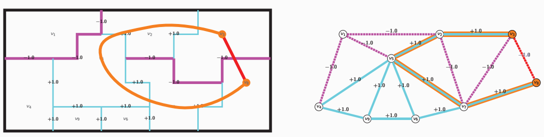


Figure: A violated cycle constraint (eq. (2)): The edge between node $v3$ and $v8$ is cut. But there is a path of uncut edge connecting $v3$ and $v8$. Violated contraints are added to the ILP in a cutting plane fashion.

Lifted Mulicut Objective [2]

- $G = (V, E)$
- larger graph $G' = (V, E')$ with $E \subseteq E'$
- edge weights $c : E' \to \mathbb{R}$ (penalize or reward precisely the decompositions of G (!) for which the nodes v and w are in distinct components)

The lifted multicut problem:

$$\min_{x \in \{0,1\}^{E'}} \sum_{e \in E'} c_e x_e \tag{3}$$

$$\text{subject to} \quad \underbrace{\forall Y \in \text{cycles}(G)\ \forall e \in Y :\ x_e \le \sum_{e' \in Y \setminus \{e\}} x_{e'}}_{\text{Ensures valid segmentation without dangling edges}} \tag{4}$$

$$\underbrace{\forall vw \in E' \setminus E\ \forall P \in vw\text{-paths}(G) :\ x_{vw} \le \sum_{e \in P} x_e}_{\text{If additional edges wv \textbf{is cut}, ensure that no path of uncut edges between u and v in } G \text{ exists}} \tag{5}$$

$$\underbrace{\forall vw \in E' \setminus E\ \forall C \in vw\text{-cuts}(G) :\ 1 - x_{vw} \le \sum_{e \in C} (1 - x_e)}_{\text{If additional edges uv \textbf{is not cut}, ensure that no cut in G exists which separates uv}} \tag{6}$$

Why Lifted Multicuts

Optimal Multicut: Optimal Lifted Multicut:

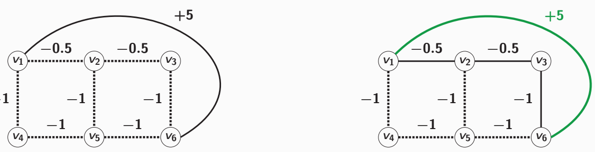


Figure: The left figure is an instance of the minimum cost multicut problem. Cut edges are depicted as dotted lines. The right figure is an instance of the minimum cost lifted multicut problem with one edge in $E' \setminus E$ depicted in green. The lifted edge with cost 5 causes the nodes $v1$ and $v6$ to be connected in G by a path of uncut edges.

Fusion Move Algorithm for Lifted Multicuts

We show how to implement a fusion moves alg.[8, 5] for the lifted multicut objective:

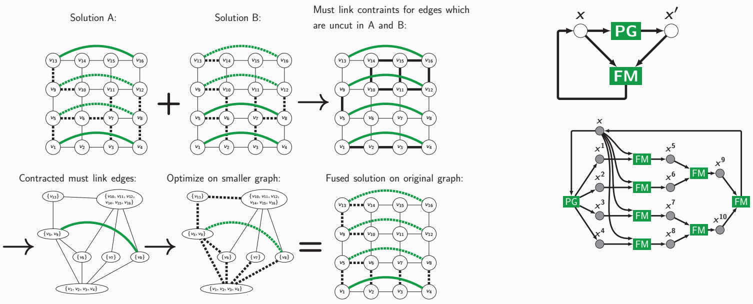


Figure: **Left Figures:** Two solutions of the lifted multicut problem A and B can be fused into a single solution by contracting all edges which are uncut in A and B (lifted edges in $E' \setminus E$ are depicted in green). We can optimize the lifted multicut problem on the smaller contracted graph to get the fused solution. **Right Figures:** In a fusion move algorithm, proposal generation (PG) and fusion moves (FM) can be combined in different ways. We implement and study serial fusion moves (top) and parallel fusion moves (bottom)

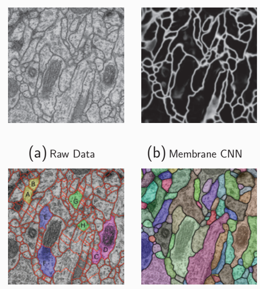
Source Code available: GitHub

`https://github.com/DerThorsten/nifty`

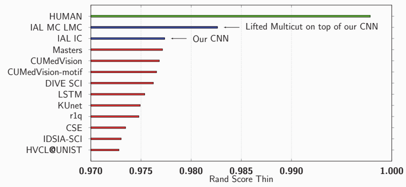
Results: ISBI Challange 2012

Segmentation of Neuronal Processes in EM Images:

- Learn membrane probabilite map with CNN
- Generate superpixels
- Learn cut-proablities for local edges (A-B, C-D) and lifted edges (E-F, G-H)
- Optimize with the proposed algorithm



(a) Raw Data (b) Membrane CNN (c) Superpixel (d) LMC Result



Complete Pipeline / Source Code available: GitHub

`https://github.com/DerThorsten/lifted_fusion_moves_eccv_2016`

Results: Benchmarks

Table: **Left**: Performance on the large and hard instances of the minimum cost lifted multicut problem of [7]. **Right:** performace on instances of a generalization of [1]

Algorithm	Objective	Time [s] (1,2,4,8 threads)
our Alg.	**−627482**	61 / 32 / 25 / 22
GAEC [7]	**−627447**	10
KLj [7]	**−627455**	121

Algorithm	Objective	Time [s] (1/2/4/8 Threads)
our Alg.	**-2.29e+07**	14.8 / 8.83/ 6.33/ 5.21
GAEC	-1.53e+07	13.8
GAEC + KLj	-2.27e+07	29.3

References

[1] Amir Alush and Jacob Goldberger. "Ensemble segmentation using efficient integer linear programming". In: *Pattern Analysis and Machine Intelligence* 34.10 (2012), pp. 1966–1977.
[2] Bjoern Andres. "Lifting of Multicuts". In: *CoRR* abs/1503.03791 (2015). http://arxiv.org/abs/1503.03791.
[3] Björn Andres et al. "Probabilistic Image Segmentation with Closedness Constraints". In: *ICCV*. 2011.
[4] Nikhil Bansal, Avrim Blum, and Shuchi Chawla. "Correlation Clustering". In: *MACHINE LEARNING*. 2002, pp. 238–247.
[5] T. Beier, F. A. Hamprecht, and J. H. Kappes. "Fusion Moves for Correlation Clustering". In: *CVPR. Proceedings, in press*. 2015.
[6] Jörg Hendrik Kappes et al. "Globally optimal image partitioning by multicuts". In: *EMMCVPR*. Springer. 2011, pp. 31–44.
[7] Margret Keuper et al. "Efficient Decomposition of Image and Mesh Graphs by Lifted Multicuts". In: *ICCV*. 2015.
[8] V. Lempitsky et al. "Fusion Moves for Markov Random Field Optimization". In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* 32.8 (2010), pp. 1392–1405.